1.	The	curve	with	eqi	uation	<i>y</i> =	f(x)) where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0 (4)$$

The iterative formula

$$x_{n+1} = \frac{1}{7} (2 + 4x_n^2 - 2x_n^3)$$

is used to find an approximate value for α .

Starting with $x_1 = 0.3$

- (b) calculate, giving each answer to 4 decimal places,
 - (i) the value of x_2
 - (ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

2.

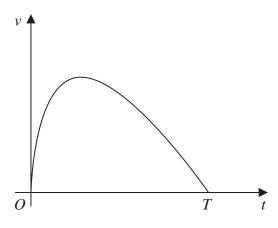


Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v \,\text{ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t) \ln(t+1)$$
 $0 \le t \le T$

where *t* seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of *T*

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t+1)} - 1 \tag{4}$$

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

- (c) (i) find the value of t_3 to 3 decimal places,
 - (ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

3. A curve has equation y = f(x), where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \qquad x > \ln \sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(b) Hence show that the *x* coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4} \tag{2}$$

The equation $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation y = x

Using Diagram 1 on next page,

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of
$$\beta$$

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)

Question 3 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

